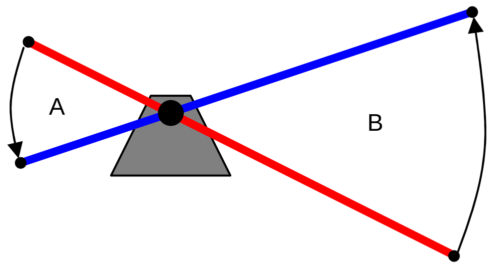
EBOT Workshop #1

Basic Mechanical Design

Part 2: Mechanical Robot Theory

Levers

Below is a simple lever, moving from the red position to the blue position. Side A is half the length of side B.

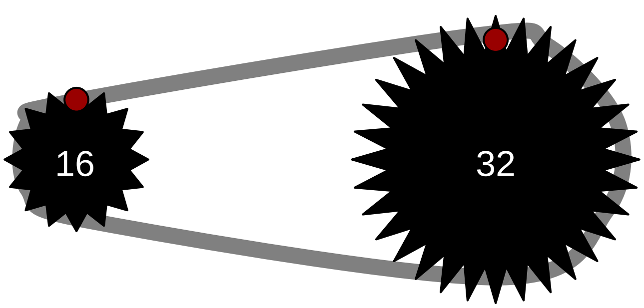


As you can see from the picture, when I lift side B of the lever from red to blue, the point at the tip of side B has to move twice as far as the point at the tip of side A.
Because the point on A is moving half the distance as the point on B in the same amount of time, it is going half the speed. However, this isn't the only difference between the two sides.

In order to lift side B, I have to do a certain amount of work, which is transferred to side A. *Work* = *Force* * *Distance*, and since the *Work* on both sides is the same but the *Distance* on side A is half what it is on side B, the *Force* on side A must be twice what it is on side B.

Chain and Sprocket Theory

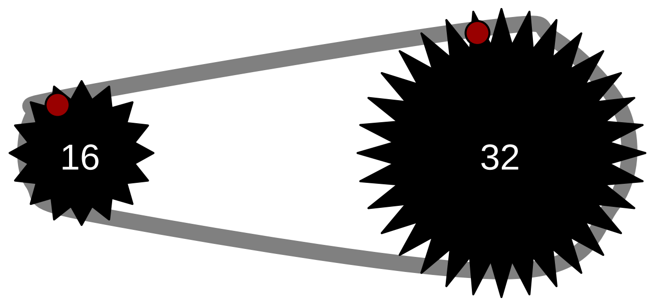
Before we get into the theory, let me explain a little about sprockets. Sprockets and gears are similar in function, but there are a few key differences. Gears mesh directly with other gears, while sprockets connect with other sprockets with chains (never try to mesh two sprockets directly together – although they look somewhat like gears, you will be disappointed with the results). Two gears connected together will rotate in opposite directions, while two sprockets connected together will rotate in the same direction. Those things on your bike are sprockets, while the things on your can opener are gears.



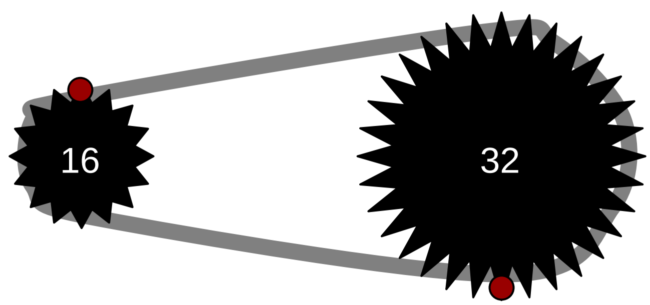
The crude diagram above represents a simple chain and sprocket system. On the right is a 16 tooth sprocket, on the left is a 32 tooth sprocket, and they are connected with a grey chain. I have marked one tooth on each sprocket with a red dot.

All sprockets are designed to fit a particular chain, and the distance between teeth of a sprocket will always be the same as the distance between links of the chain it was designed for. For simplicity, the Robovation kit only includes one size of chain, and therefore all the sprockets will work with the chain.

If I rotate one sprocket forward one tooth (for example, if I turned the 16 tooth sprocket one-sixteenth of a turn), I will be advancing the chain by one link. Because the distance between the teeth of all the sprockets is the same as the distance between the links, advancing the chain one link will advance the other sprocket by one tooth (shown below).



Following this forward, if I attach a motor to the 16 tooth sprocket and rotate it through a complete revolution, I will advance both sprockets by 16 teeth. Because the larger sprocket has 32 teeth, it has only done half a revolution in the save time that the smaller sprocket did a complete revolution. In other words, the 32 tooth sprocket went half the speed as the 16 tooth sprocket (and an output shaft connected to the 32 tooth sprocket will go half the speed of the input shaft connected to the motor)



Like with the levers above, speed isn't the only difference between the two sprockets. The motor did a certain amount of work in turning the input shaft a complete revolution, and that work was transferred to the output shaft. However, because the output shaft only completes half a revolution, that work had to be compressed into the smaller amount of rotation. The *Work = Force * Distance* equation from the lever, when translated to work with rotating objects, becomes *Rotational Work = Torque * Angle*. Just like with levers, if *Work* stays the same, and *Angle* is halved, *Torque* must double.

What does this all mean? That in the above example, the output shaft will spin at half the speed of the input shaft, but will have twice the torque (which basically means it can do twice as much force on anything you attach to the shaft).

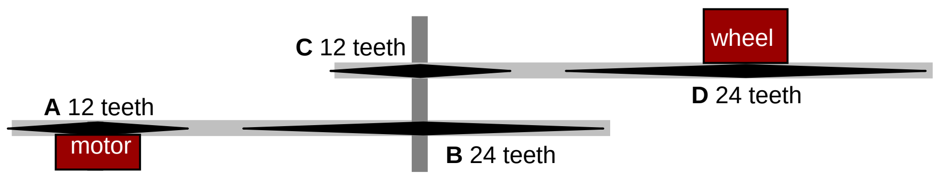
You can also see that if I attached a motor to the 32 tooth sprocket, that the output shaft attached to the 16 tooth sprocket will go at twice the speed, but with half the torque.

This all boils down to the following equations:

$$rpm_{output} = rpm_{input} * \frac{teeth_{input}}{teeth_{output}}$$

$$torque_{output} = torque_{input} * \frac{teeth_{output}}{teeth_{input}}$$

It is recommended that you never have a ratio between two sprockets that is greater than 1:3. For example, a 24 tooth sprocket should only be connected though a chain to a sprocket with between 8 and 72 teeth. However, it is possible to get ratios larger than 1:3 using sprockets, as shown below:



Let's pretend the motor is spinning at 4 RPM. Using the above equation with A as the input and B as the output shows that B (and therefore the shaft) is spinning at 2 RPM. However, the shaft serves as both the output for B and the input for C, so we use the equation again with 2 RPM as the input speed, C as the input, and D as the output. Now, the equation shows that the speed of the output at D (and therefore the wheel) is 1 RPM. Comparing the motor speed (4 RPM) and the wheel speed (1 RPM), we see that we have achieved a 4:1 ratio!

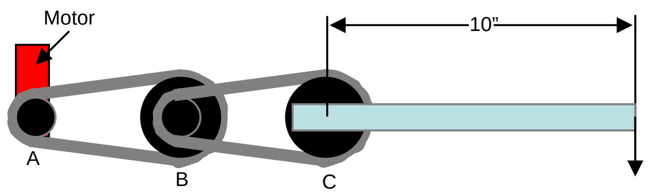
You can stack as many chain and sprocket systems as you wish, but remember that you lose about 3% efficiency with each stage.

It is also important that you consider the strength of the chain in your designs. The maximum working load of the chain included in the kits is 6 pounds (about 100 ounces). Although the chain will usually not break until 9 to 12 pounds of force are applied to it, it is highly recommend that your design not exceed the maximum working load.

To calculate the load on your chain, you will need one more equation:

$$force = \frac{torque}{radius}$$

Let's look at the example of the following arm with a two stage reduction. The arm is 10" long, the large sprockets are 1" diameter, and the small sprockets are .5" diameter (and have half as many teeth as the large sprockets). There is a 6 ounce weight on the end of the arm.



The 3 ounce weight is going to exert a force of 3 ounces on the end of the lever. We can use the above equation to figure out the torque on the shaft at C. Using 3 ounces as the force and 10 inches as the radius, *3 = torque / 10*, so the torque is 30 ounce inches. We can then use the same equation to figure out the force being applied to the chain running between B and C, this time with 30 oz*in as the torque and .5" as the radius. Now, *force = 30 / .5*, so the force on the chain is 60 ounces, which is within the tolerances.

Now, lets look at the chain between A and B. To figure out the torque on shaft B, we use the torque ratio equation from before:

$$torque_{output} = torque_{input} * \frac{teeth_{output}}{teeth_{input}}$$

Using 30 ounce inches as the output torque, and 2 for the radio of teeth, we find that the input torque (the torque on shaft A) is 15 ounce inches. Using the force and torque equation, with 15 ounce inches of torque and a radius of .5", we find that the force on the chain between A and B is 30 ounces, which is well within tolerances.

You can also use the same equations to figure out the maximum load an arm like this can hold, which I will leave as an exercise for the reader.

Robot Speed

If you know the speed of your wheels in rotations per minute and the diameter of your wheels in inches, you can use the following equation to figure out the speed of your robot in feet per second:

$$speed_{robot} = \frac{rpm_{wheel}}{60} * \frac{Diameter_{wheel}}{12} * \pi$$

But how do we know the speed of our wheels? Unless we simply want to measure and use trial and error, we must look at the motor performance data. The data for the motors in the Robovation kit is shown below:

Robovation Motor Performance Data					
Speed (RPMs)	**Torque (oz. in.)**	**Current (Amps)**	**Power Out (Watts)**	**Efficiency**	**Heat (Watts)**
170	0.00	0.1	0.0	0%	1
159	4.68	0.3	0.5	26%	2
147	9.35	0.4	1.0	34%	2
136	14.03	0.5	1.4	36%	2
125	18.71	0.6	1.7	36%	3
113	23.38	0.7	2.0	34%	4
102	28.06	0.8	2.1	32%	4
91	32.73	0.9	2.2	29%	5
79	37.41	1.0	2.2	26%	6
68	42.09	1.1	2.1	23%	7
57	46.76	1.2	2.0	19%	8

There are a couple of interesting things to note here. First of all, the slower the speed of the motor, the more torque it produces. This can be a bit misleading. This does not mean that you can simply program your motor to go slower to get more torque! If you program the motor to go at less than full speed, you will get much less torque than you would at full speed (since the speed controller in the motor essentially works by turning the motor on and off rapidly). What the data in the chart means is that when the motor has no load on it, it spins at 170 RPM but produces no torque (since the force applied to the motor and the force the motor applies back must be equal when the motor is running at constant speed). If you applied 14 ounce inches of torque to the motor, it would slow to 136 RPM and produce 14 ounce inches of torque back. This table only goes up to 57 RPM and 46.76 ounce inches because applying any more torque than that to the motor can will cause it to overheat and could possibly damage the gears inside.

For calculating the speed of the robot, we assume that the motor is performing at peak efficiency (which on these motors is occurs somewhere between 120 and 140 RPM). To make calculations easier, lets use 120 RPM. Lets also say that we want our robot to go 3 feet per second (a good number for a robot of this scale). Assuming Pi is exactly 3, the robot speed equation becomes:

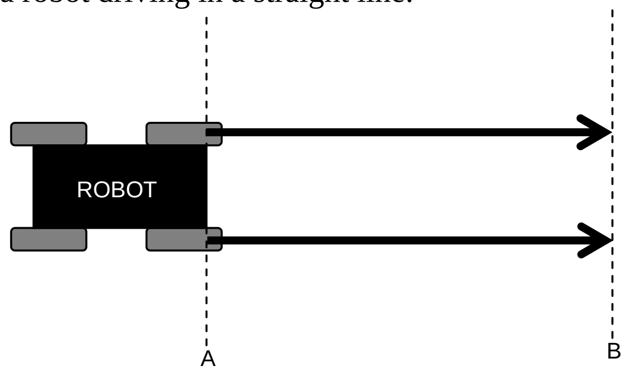
$$3 = \frac{\sim 120}{60} * Diameter_{wheel} * \sim 3$$

And we can solve that we need 6 inch diameter wheels.

Now, let's say that 6 inch wheels wouldn't fit on our robot, and we wanted to use 3 inch wheel instead. The equation shows that our robot would only go 1.5 feet per second, which is too slow. However, if we put a sprocket and chain system between the motor and the wheel that has a ratio of 1:2, we can increase rpm_{wheel} from 120RPM to 240RPM, which means our robot will now go 3 feel per second again!

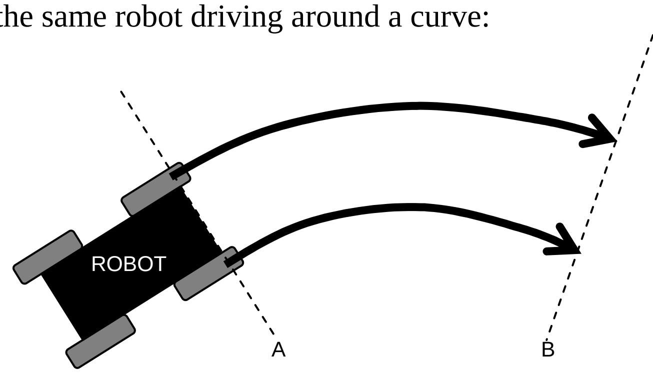
Skid (Tank) Steering

Unlike cars, most small robots are steered using skid steering. To understand how this works, let's look at a robot driving in a straight line:



As the robot travels from A to B, the wheels on both sides travel the same distance in the same time (and therefore go the same speed).

Now, let's look at the same robot driving around a curve:

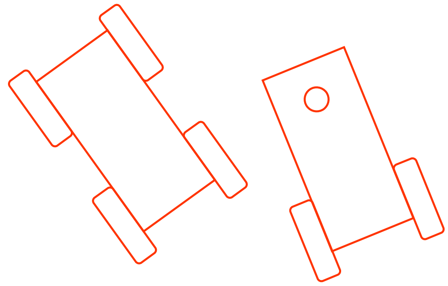


As the robot travels from A to B, the outside wheels have to travel a further distance in the same amount of time that the inner wheels travel a shorter distance, and therefore the outer wheels are turning faster than the inner wheels.

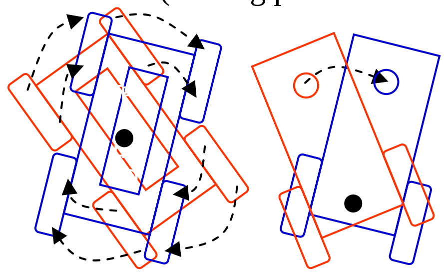
Therefore, on a robot with fixed wheels, driving one set of wheels faster than the other will make the robot turn towards the slower moving wheels. This is a much simpler system than the rack-and-pinion steering in your car, and it allows the robot to do things like spin in place (by driving one side forwards and the other side backwards).

4 Wheels vs. 2 Wheels

Below are a sample 4 wheel and 2 wheel robots:



Now watch what happens as each turns (starting position is red, ending position is blue):



Each robot turns around its black dot. As the 4 wheeled robot turns, each wheel has to move sideways, as shown by the arrows. Because wheels are designed not to slip, this sideways motion is difficult, and causes your robot to turn slowly (and may cause your wheels to fall off, if they are not properly held in with shaft collars).

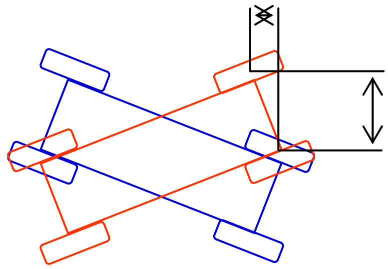
As the 2 wheel robot turns, its wheels have very little, if any, sideways motion (assuming that the center of gravity of the robot is close to the two wheels). The circle in the front of the robot is a skid of some soft (this is any smooth piece of plastic, such as a ping-pong ball or a bottle cap). Because this skid is smooth, it doesn't mind making the large sideways motion, and the robot will turn easily.

However, the 2 wheeled robot does have disadvantages. The skid creates drag, so the robot may have trouble driving on some surfaces. However, with robots this small and light, that is usually not a problem. Also, most robots with skids are unable to climb onto platforms or over most obstacles.

The choice to do two of four wheels is one that every team will have to make based on the tasks the robot will have to do. However, we would not recommend building a 4 wheel robot unless all four wheels are driven (either with one motor per side connected with chain and sprockets or one motor per wheel).

Wheel Base

Two important factors to consider when building your robot are how wide and how long your chassis will be. To understand why this matters, lets look at a long skinny robot (this robot has four wheels, but the same principles apply to two wheel robots as well).

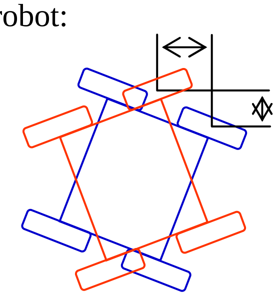


You will notice that as the long and skinny robot turns, the forward-backward motion of the wheels is very small, but the sideways motion is very large.

The short forward-backward motion of the wheels acts just like the short part of a lever, which means you need more force to turn through a certain angle than you would on a robot with more forward-backward motion while turning through the same angle. This means it is easier to go straight, but harder to turn.

The long sideways motion means that the wheels have to do a lot of sideways rubbing. As we discussed above, sideways wheel motion is difficult, and the more sideways motion there is, the more force is needed to move the robot sideways. If you get too much sideways wheel motion, your robot may even start jumping and hopping when it tries to turn.

Now let's look at a short and fat robot:

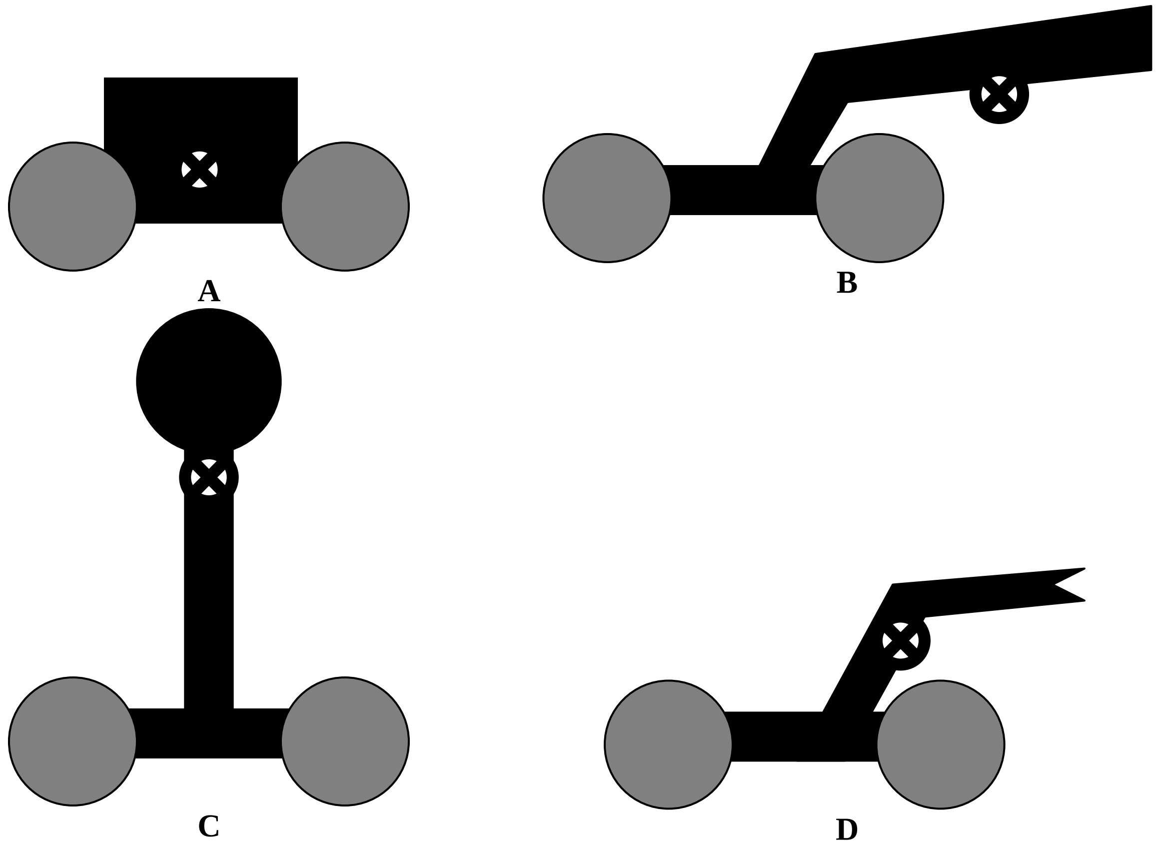


You will notice that the forwards-backwards motion is now larger than the sideways motion, which means that it will be easier to turn and there won't be as much rubbing, but it will be much harder to drive the robot in a straight line and, since it will turn faster, harder to control.

The ideal robot is a trade-off between easy turning and easy straight travel. The balance also depends on what your robot is being designed to do (some robot designs may never need to turn, while others may never need to travel straight).

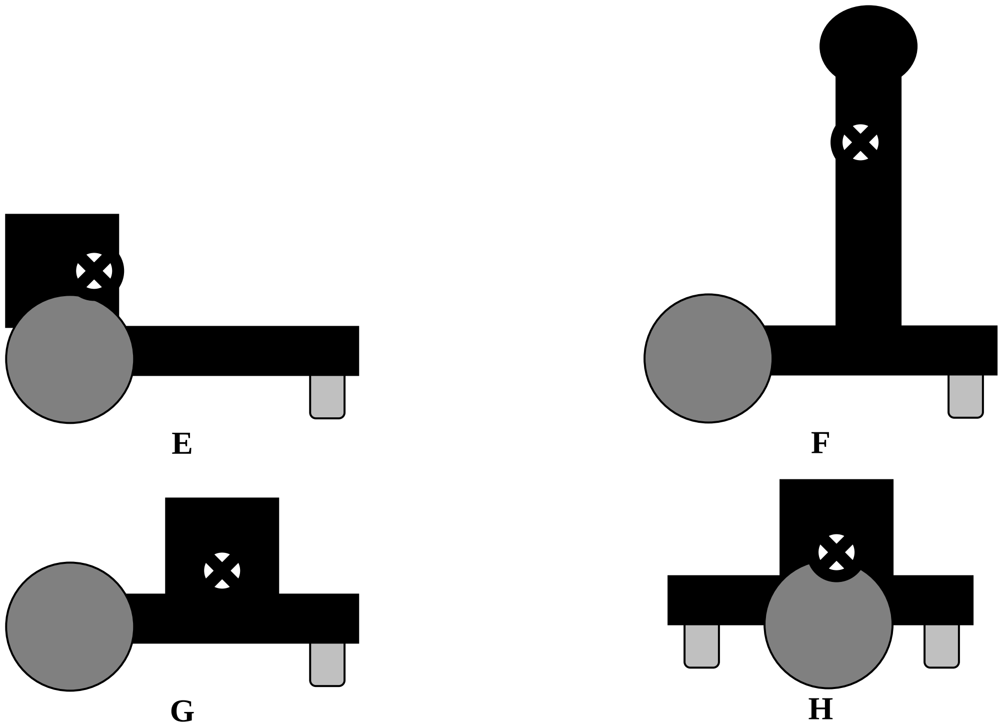
Center of Gravity

Below I have four sample four wheeled robots. The center of gravity is marked by the X.



Of these, both A and D are okay, because the center of gravity is between the two wheels. In D, most of the weight is over the front wheels, but this is okay if the front wheels are driven by the motor. In B, the center of gravity is too far forward, and the robot will probably do lots of wheelies (which, while cool, is probably not the intention of the robot). In C, although the center of gravity is over the wheels, the height of the center of gravity will make the robot likely to tip over if it has to go up a slope.

Below I have four sample two wheeled robots. The center of gravity is marked by the X.



Of these, both E and H are okay, because the center of gravity is between the skid and the wheels, but mostly over the wheels. The closer the center of gravity is to the skid, the more weight is supported by the skid, and the more friction you will get, which is why G is not a very good design. In H, we used two skids with a wheel in the middle, which is a good design but requires very accurate adjustment of the skids so that the wheels touch the ground, but the robot doesn't see-saw too much when it changes direction. F is not good because the center of gravity is too high and it is over the center of the robot.